solution with a concentration $c = 5\cdot 10^{-3}$ kg/m³ the average absolute value of the transverse pulsation is decreased by 1.7 times.

NOTATION

N, particle concentration; w_{av}, average velocity in transverse direction; t, time; s, distance between magnetic marking coils; v_{av}, average velocity in longitudinal direction; M, magnetization per unit volume of liquid; I, intensity of NMR signal; Re, Reynolds number; k, $\varkappa$, proportionality factors.

LITERATURE CITED

1. Yu. F. Ivanyuta and L. A. Chekalova, "Turbulent flows of polyoxyethylene solutions in a pipe with large-scale surface roughness," Inzh.-Fiz. Zh., 31, No. 2, 225-230 (1976).
2. E. S. Mikhailova and E. M. Khabakhpasheva, "Boundary turbulence in the flow of a weak solution of polyacrylamide," in: Experimental Study of the Structure of Boundary Turbulence and the Viscous Sublayer [in Russian], Sib. Otd., Inst. Teplofiz. Akad. Nauk SSSR, Novosibirsk (1976), pp. 58-64.
3. F. A. Seyer and A. B. Metzner, "Turbulence phenomena in drag reducing systems," AIChE J., 15, 426-430 (1969).
4. J. W. Hoyt, "Influence of admixtures on frictional resistance in a liquid," Teor. Osn. Inzh. Raschet., 94D, No. 2, 1-22 (1972).
5. A. I. Zhernovoi and G. D. Latyshev, Nuclear Magnetic Resonance in a Flowing Liquid [in Russian], Atomizdat, Moscow (1964).
6. A. I. Zhernovoi and A. E. Kokin, "A method of studying turbulent diffusion in a liquid stream in a pipe," Inventor's Certificate No. 600449 with priority of Dec. 17, 1975, Byul. Izobr., No. 12, 170 (1978).

BOUNDARY LAYER ABOVE A SEMI-INFINITELY LARGE HOT PLATE IN A MEDIUM WITH PHASE TRANSITION

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A self-adjoint solution in the "boundary layer" approximation is constructed to the problem of streamlining of a hot plate by a medium in which a phase transformation occurs. A longitudinal pressure gradient exists within the region of the liquid phase.

Let a substance in the solid state with density ρ_0 and phase transition temperature T_f move in the direction of the x axis at a constant velocity U_0 in the half-space $y > 0$. The temperature of the solid at $y \to \infty$ is given and assumed to be $T_\infty < T_f$. At $y = 0$ there is located a semi-infinitely large stationary flat plate ($0 \leqslant x < \infty$) whose temperature is everywhere the same $T_W > T_f$. Above this plate, moreover, there forms a layer $0 < y < y_f(x)$ (where $y_f(x)$ denotes the phase transition surface) within which the liquid phase of the given substance flows (Fig. 1).

The processes of heat and mass transfer within the region of the liquid phase $0 < y < y_f(x)$, $x > 0$ are described by the system of equations [1]

$$\frac{\partial\Psi}{\partial y}\frac{\partial\Delta\Psi}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial\Delta\Psi}{\partial y}=\nu\Delta\Delta\Psi, \tag{1}$$

$$\frac{\partial\Psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial T}{\partial y}=\varkappa\Delta T+\frac{\nu}{c}\left[4\left(\frac{\partial^2\Psi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\Psi}{\partial y^2}-\frac{\partial^2\Psi}{\partial x^2}\right)\right].$$

Here x, y are respectively the longitudinal and the transverse coordinates, $\Psi = \Psi(x, y)$ is the flow function ($u = \partial\Psi/\partial y$, $v = -\partial\Psi/\partial x$ are respectively the longitudinal and the transverse

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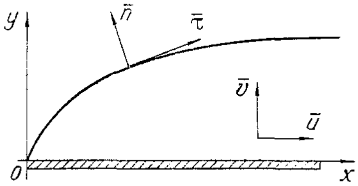

Fig. 1. Flow pattern.

velocities), and $\nu, \varkappa, \rho$, c are respectively the kinematic viscosity, thermal diffusivity, density, and specific heat of the substance in the liquid state.

The equations of transfer simplify for the region outside the liquid phase, because here the velocity of the medium is known: ($\partial\Psi/\partial y = U_0$ and $\partial\Psi/\partial x = 0$) so that

$$U\frac{\partial T}{\partial x} = \varkappa_0 \Delta T, \tag{2}$$

where $\varkappa_0$ denotes the thermal diffusivity of the substance in the solid state.

The system of Eqs. (1), (2) must be supplemented with conditions at the isothermal plate y = 0, as $y \to \infty$, x > 0, and at the phase transition surface $y = y_f(x)$, x > 0.

Owing to "adhesion" of the liquid to the solid surface, the components of velocity in the direction of vector τ tangent to the phase transition surface $y = y_f(x)$ should be equal on both sides of that surface (Fig. 1). As a consequence,

$$\frac{\partial\Psi}{\partial y}(x, y_f - 0) - \dot{y}_f \frac{\partial\Psi}{\partial x}(x, y_f - 0) = \dot{y}_f U_0, \quad \dot{y}_f = \frac{dy_f}{dx}, \tag{3}$$

where ±0 denotes that the corresponding limit has been reached.

From the continuity of the stream of substance through the phase transition surface, furthermore, there follows the condition

$$\dot{y}_f \frac{\partial\Psi}{\partial y}(x, y_f - 0) + \frac{\partial\Psi}{\partial x}(x, y_f - 0) = \beta \dot{y}_f U_0, \quad \beta \equiv \frac{\rho_0}{\rho}. \tag{4}$$

Taking into account relations (3) and (4), also considering that the plate can be permeable, one can now write all the boundary conditions which the sought functions $\Psi(x, y)$ and $T(x, y)$ must satisfy

$$\begin{gathered}
\frac{\partial\Psi}{\partial y}(x, 0) = 0, \quad \frac{\partial\Psi}{\partial x}(x, 0) = \delta(x), \quad T(x, 0) = T_w. \\
\frac{\partial\Psi}{\partial y}(x, y_f - 0) = U_0 \frac{1 + \beta \dot{y}_f^2}{1 + \dot{y}_f^2}, \quad \frac{\partial\Psi}{\partial x}(x, y_f - 0) = U_0 \frac{(\beta - 1)\dot{y}_f}{1 + \dot{y}_f^2}, \\
T(x, y_f) = T_f, \quad \dot{y}_f \frac{\partial T}{\partial x}(x, y_f - 0) - \frac{\partial T}{\partial y}(x, y_f - 0) = \frac{\beta\lambda}{c\varkappa} \dot{y}_f U_0 + \\
+ \frac{c_0 \rho_0 \varkappa_0}{c \rho \varkappa}\left[\frac{\partial T}{\partial x}(x, y_f + 0)\dot{y}_f - \frac{\partial T}{\partial y}(x, y_f + 0)\right], \quad T(x, \infty) = T_\infty.
\end{gathered} \tag{5}$$

Here λ is the latent heat of fusion. We note that the condition before the last one of conditions (5) follows from the expression for the thermal flux to surface $y = y_f(x)$ written in a form which takes into account the phase transition occurring at that surface.

The system of Eqs. (1), (2) with conditions (5) completely define the boundary-value problem, which we will solve using the approach proposed by N. E. Kochin for solving the Blasius problem [2].

An examination of the boundary-value problem (1), (2), (5) indicates that its solution can be sought in the form of a power series with respect to the independent variable x

$$\Psi(x, y) = \nu \sum_{k=0}^{\infty} f_k(\eta) (U_0 x/\nu)^{(1-k)/2} / \alpha,$$

$$T(x, y) = T_w - (T_w - T_f) \sum_{k=0}^{\infty} \theta_k(\eta) (U_0 x/\nu)^{-k/2},$$

$$y_f(x)=\frac{\nu}{U_0}\sum_{k=0}^{\infty}y_k\,(U_0x/\nu)^{(1-k)/2},\quad \eta=y/y_f(x),$$

where α is an unknown constant most expediently assumed equal to y_0. With these expressions inserted into the system of Eqs. (1), (2) and conditions (5), the problem reduces to boundary-value problems in successive approximations involving ordinary differential equations. We will consider only the zeroth approximation, which corresponds to the conventional approximation of a boundary layer. Within the region of the liquid phase, then, the original problem reduces to

$$2f'''+ff'=2\gamma, \tag{6}$$

$$2\theta''+\sigma f\theta'=2mf''^2, \tag{7}$$

$$f'(0)=0,\ f(0)=0,\ \beta f'(1)-f(1)=0,\ f(1)=\alpha^2\beta, \tag{8}$$

$$\theta(0)=0,\ \theta(1)=1,\ \theta'(1-0)=\alpha^2d+\xi\theta'(1+0). \tag{9}$$

Here $\sigma=\nu/\varkappa$; $m=U_0^2/c\alpha^4(T_W-T_0)$; $d=\beta\lambda\sigma/2c(T_W-T_f)$; $\xi=c_0\varkappa_0\rho_0/c\varkappa\rho$; ω, injection parameter; γ, dimensionless constant which determines the pressure gradient $\partial p/\partial x=\gamma\rho U_0^2/\alpha^4x$; the prime sign denotes differentiation with respect to the self-adjoint variable η. The form of function $\delta=\delta(x)$, which determines the pattern of injection through the porous surface $y=0$, is stipulated by the condition of self-adjointness as $\delta(x)=-\omega\sqrt{\nu U_0}/2\alpha\sqrt{x}$ ($\omega>0$ corresponds to suction, $\omega<0$ corresponds to injection).

Within the region of the solid phase, on the other hand, the problem reduces to

$$2\theta''+\sigma\alpha^2\eta\theta'=0; \tag{10}$$

$$\theta(1)=1;\ \theta(\infty)=(T_w-T_\infty)/(T_w-T_f). \tag{11}$$

Assuming that $T_f-T_\infty\ll T_W-T_f$, we can let $\theta(\infty)=1$ in the last of conditions (11). Then the solution to problem (10), (11) is $\theta(\eta)\equiv1$. For this reason, it is necessary to let $\theta'(1+0)=0$ in the last of conditions (9).

With the aid of function $f=f(\eta)$ found by solving the problem (6)-(9) one can now determine

$$\Psi=\sqrt{\nu U_0x}\,f/\alpha,\ u\equiv\partial\Psi/\partial y=U_0f'/\alpha^2,\ v\equiv-\partial\Psi/\partial x=\sqrt{\nu U_0}\,(f'\eta-f)/2\alpha\sqrt{x}.$$

With constant γ assumed to be known, the hydrodynamic problem (6), (8) becomes independent of the thermal problem. Moreover, the last of conditions (8) can be used for determining the quantity $\alpha=\sqrt{f(1)/\beta}$. It is expedient to seek $f=f(\eta)$ in the form of a power series with respect to the self-adjoint variable $f=\sum_{k=0}^{\infty}a_k\eta^k$, Eq. (6) then yielding for a_k the recurrence relation $a_3=(\gamma-a_2\omega)/6$ and $a_k=-\sum_{j=0}^{k-3}a_ja_{k-1-j}(k-j-1)(k-j-2)/2k(k-1)\cdot(k-2)$ $(k>3)$. The remaining unknowns a_0, a_1, a_2 must be found from the first three of boundary conditions (8): $a_0=\omega$, $a_1=0$, $\sum_{k=0}^{\infty}(\beta k-1)a_k=0$. (The last of these equations must be solved for a_2, which can be done, e.g., by the method of successive approximations. As the zeroth approximation can serve $a_2^0=(\omega-\gamma/3)/(2\beta-1-\omega/3)$).

After $f=f(\eta)$ has been determined, problem (7), (9) must be solved for $\theta=\theta(\eta)$ and conditions (9) also yield the quantity d

$$\theta=\int_0^{\eta}e^{-F}\left[\alpha^2d+\int_1^{\eta}mf''^2e^Fd\eta\right]d\eta,$$

$$d=\left\{1-\int_0^1e^{-F}\left[\int_1^{\eta}mf''^2e^Fd\eta\right]d\eta\right\}/\alpha^2\int_0^1e^{-F}d\eta,$$

where

$$F=\frac{\sigma}{2}\int_1^{\eta}fd\eta.$$

It is evident that in the general case both heat and mass transfer within the given boundary layer depend on five parameters: σ, m, d, ω, β. Numerical calculations according to the relations derived here are sufficiently elementary for programming on a computer. We

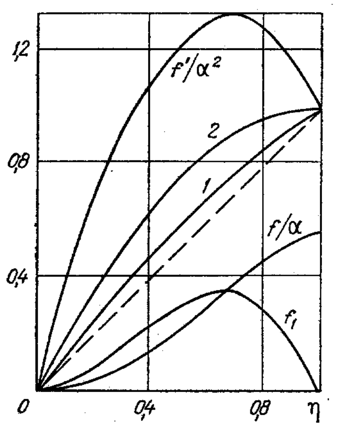

Fig. 2. Basic flow characteristics

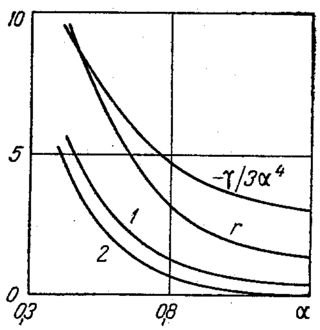

Fig. 3. Relations between flow parameters and physical characteristics of the problem.

will, therefore, consider only the simplest case $m = \omega = 0$ and $\beta = 1$. The relations $f(\eta)/\alpha$, $f'(\eta)/\alpha^2$, and $f_1 = 2(f'\eta - f)/\alpha$ have been plotted in Fig. 2 for $\sigma = 5$ and $d = 1.98$, which roughly corresponds to a water—ice system for $T_w = 373°K$. There a $\theta = \theta(\eta)$ dependence is formed for the cases $\sigma = 5$ and $d = 1.98$ (curve 1) or 0.06 (curve 2). The dashed line corresponds to small values $\sigma \ll 1$, at which function θ is linear ($\theta \approx \eta$).

The graph in Fig. 3 depicts the dependence of parameter α, determining the size of the molten zone on the parameters $-\gamma/3\alpha^4$ and $r = f''(0)/\alpha^3$, respectively, determining the pressure gradient and the viscous stress at the plate, both single-valued functions of parameter α. On the same diagram is also shown the dependence of α on the physical parameters of the problem. Curves 1 and 2 correspond to $\sigma = 0.5$ and 5.0, respectively.

In conclusion, we note that a boundary layer localized in space also forms in the case of a semi-infinitely largeplate streamlined by a nonlinearly viscous dilatant non-Newtonian fluid [3]. It must be emphasized, however, that the principal feature of a boundary localized in space as in this particular case is existence in it of a nonzero pressure gradient. Exactly this pressure gradient makes it possible to satisfy the condition of a zero transverse velocity component at the phase transition surface.

LITERATURE CITED

1. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Nauka, Moscow (1973).
2. N. E. Kochin, Collection of Works [in Russian], Vol. 2, Izd. Akad. Nauk SSSR, Moscow—Leningrad (1949).
3. K. B. Pavlov, "Theory of the boundary layer in nonlinearly viscous non-Newtonian media," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3, 26-33 (1978).